Vibration and Buckling Analysis Of Cracked Beam-Columns

A Thesis Submitted In Partial Fulfillment
of the Requirements for the degree of

Master of Technology
In
Civil Engineering
(Structural Engineering)

By

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NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA
Rourkela-769008,
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CERTIFICATE

This is to certify that the thesis entitled, **"VIBRATION AND BUCKLING ANALYSIS OF CRACKED BEAM-COLUMNS"** submitted by **Miss. Pratima Sabar** in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Civil Engineering** with specialization in **Structural Engineering** at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

Date: May 30, 2008

Place: Rourkela

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ACKNOWLEDGEMENTS

I express my gratitude and sincere thanks to **Prof. U.K.Mishra,** for his guidance and constant encouragement and support during the course of my work in the last one year. I truly appreciate and value his esteemed guidance and encouragement from the beginning to the end of this thesis, his knowledge and company at the time of crisis remembered lifelong.

My sincere thanks to **Prof K. C. Patra,** Head of the Civil Engineering Department, National Institute of Technology Rourkela, for his advice and providing necessary facility for my work

I am also very thankful towards **Prof. Asha Patel,** Assistant Professor, my faculty adviser and all faculty members of structural engineering specialization for their help and encouragement during the project.

I also thank all my friends who have directly or indirectly helped me in my project work and in the completion of this report.

Finally yet importantly, I would like to thank my parents, who taught me the value of hard work by their own example. I would like to share this moment of happiness with my father, mother. They rendered me enormous support during the whole tenure of my stay in NIT Rourkela.

Pratima Sabar

CONTENTS

ABSTRACT

An important task for engineers is to determine the effect of the damage like transverse cracks on the stability characteristic of structures. The presence of cracks causes changes in the physical properties of a structure and its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. The buckling load is one of the important parameter for stability of a structural member. The present work is aimed at finding the buckling load of a cracked beam-column with a single edge crack.

Finite element method is adopted for the dynamic analysis of the beam-column. Additional flexibility coefficients of the cracked beam element are computed using 6-point Gauss quadrature and theories of fracture mechanics. Flexibility coefficients of an intact element are added to the additional flexibility matrix to get the total flexibility matrix of the element. Over all flexibility matrix of the cracked element is then obtained from the total flexibility matrix. Stiffness matrix of the cracked element is derived from the overall flexibility matrix of the element for the analysis of an Euler beam-column of mild steel with cantilever end conditions. The first four natural frequencies and the corresponding mode shapes of vibration are obtained by dynamic analysis solving the eigen value problem using a FORTRAN code. The computed results are compared with previous studies and the present formulation is established. Buckling load is then obtained solving another eigen value problem of stiffness matrix and stress matrix of the beam-column. Variation of buckling load with respect to relative crack depth, position of the cracked section and different length of span are studied. It is found that the presence crack weakens the member by decreasing the frequencies of vibration as well as buckling load. How ever this effect is more pronounced when the crack is near to the fixed end than free end. For small cracks with relatively lesser RCD value the decrease of buckling strength is less but for bigger cracks with higher RCD value the buckling strength decreases rapidly.

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NOMENCLATURE:

$[M]$= Consistent mass matrix

$[K]$ = Bending stiffness matrix of the beam

$[K_g]$= Geometric stiffness matrix

P_{cr} =Critical load

ω_n = frequency

a_1 = deflection at node 1
a_2 = slope at node 1
a_3 = deflection at node 2
a_4 = slope at node 2
L = length of the element

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ = constants

$a(x)$ =nodal displacement component

$[N]$ = Shape function matrix

$[B]$ = strain displacement matrix

RCD = Relative crack depth

E = young's modulus of elasticity

I = moment of inertia of the section with respect to *z*-axis

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks

Π_C = total potential energy of the body

G = the strain energy release rate

A_C = the effective cracked area.

$E' = E$ (for plane stress)

C_{ij} = overall additional flexibility matrix

a/h = relative crack depth

ν = poission's ratio

C_i = Weight factor

n = no. of points used for Gaussian quadrature

CHAPTER-1

INTRODUCTION

1.1 INTRODUCTION

The cracks in a structural member may develop from flaws due to applied cyclic loads, mechanical vibrations, aerodynamic loads, rocket fuel exhaust or acoustical fatigue. In civil engineering, structures like beam columns, bridges, piles, etc will bear damages due to long-term service, collision, impact, etc. An important task of engineers is to determine the effect of these damages on the stability characteristic of these structures. The presence of cracks causes changes in the physical properties of a structure which in turn alter its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. Irregular variations in the measured vibration response characteristics have been observed depending upon whether the crack is closed, open or breathing during vibration, the degree of severity and modal type. These variables consequently affect the effectiveness of structural integrity assessment.

Members that are subjected to both bending and axial compression are beam-columns. Bending is caused by either moments applied to the ends of the member or it may be due to transverse loads directly acting on the member. Extensive studies have been done on the free vibration analysis of cracked beams and elastic stability of un-cracked columns. However, vibration and buckling analysis of a cracked beam-column have been studied only by a few researchers. The study of vibration and buckling load of a slender beam-column with crack is a problem of practical interest and finds applications in aerospace, mechanical and civil engineering.

Buckling load is defined as the load at which a structural member becomes laterally unstable leading to collapse of it. It can be observed by sudden bending, warping, curling or crumpling of the elements or members under compressive stresses. The buckling of a beam-column depends upon flexural rigidity. For a homogeneous and isotropic beam-column it is in a direction perpendicular to the axis, about which the moment of inertia of the section is minimum.

Buckling of beam-column is an instability phenomenon where the change of equilibrium state from one configuration to another occurs at a critical compression value. Since the transition of this state of equilibrium is always related to motion, it is appropriate to use the dynamic analysis for the investigation. Therefore the buckling analysis of the cracked beam-column can be reduced to an eigen value problem similar to free vibration analysis of beams.

So a comprehensive literature study covering the above aspects of the beam-column is carried out.

CHAPTER-2

LITERATURE REVIEW

2.1 LITERATURE REVIEW

The fact that a crack or local defects affect the dynamic response of a structural member was known long ago. In the past, various researchers conducted investigations into the vibration and buckling characteristics of cracked structures. Kevin et al. (2000) investigated vibration and stability characteristics of a cracked beam translating between fixed supports. They developed equations of motion for the beam using Hamilton's principle and fracture mechanics throughout the analysis, the crack was assumed to be open and shallow. Skrinar (2000) implemented a simple computational model to predict buckling load of slender beam type structures with transverse crack. He obtained buckling load by the numerical solution of the governing equations and also by using a new beam-column element in finite element method. Some details of the earlier works on vibration of cracked structures were reported by Dimarogonas (1996). Later S. C. Fan and D. Y. Zheng (2002) reported about the works done by various researchers on vibration and buckling aspect of beam-columns. They mentioned about the development of the modified Fourier series (MFS) and the modified beam vibration functions (MBVF) to solve the vibration and stability problem of structures with stepped cross-sections and/or intermediate supports. They did vibration and stability analysis of cracked Euler beams and Timoshenko beams successfully using modified Fourier series (MFS). They also developed a new method for computing the buckling load reduction of a Timoshenko beam-column with an arbitrary number of transverse open cracks. They solved standard linear eigen value equations, rather than non-linear algebraic equations, in their method. Wang (2004) presented the stability analysis of a cracked beam subjected to a follower compressive load. The vibration analysis on such cracked beam is conducted to identify the critical compression load for buckling instability. Kisen et. al (2004) studied fracture behavior of cracked beam-columns with different load eccentricities. They used a beam-column element proposed by Tharp in a finite element code to study the behavior of cracked beam-columns subjected to axial and lateral loads, they also predicted the failure load of cracked columns for different crack depths and slenderness ratios. They found that the critical load of columns decreases with the presence of cracks and this decrease is small for short cracks and columns. Jiki (2007) proposed how to use Liapunov's functional method to find critical load for pre-cracked thin walled beam-columns rather than solving a set of differential equations. He obtained expression for reduced buckling strength due to edge cracks using Zubov's stability theorem and appropriate eigen value inequalities. Monsalve et. al (2007) studied stability and free vibration analysis of a Timoshenko beam-columns with a crack along its span, they modeled the crack as an intermediate flexural connection of zero length with rotational discontinuity and identical lateral deflection between two sides of the crack, they also studied the coupling effect of shear and bending deformation along the span of the member.

The present work is aimed at finding the buckling load of a beam-column with a single edge crack using finite element analysis in a FORTRAN code.

CHAPTER-3

CRACK THEORY

3.1 PHYSICAL PARAMETERS AFFECTING DYNAMIC CHARACTERISTICS OF CRACKED STRUCTURES:

Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of a crack in structures modifies its dynamic behavior. The following aspects of the crack greatly influence the dynamic response of the structure.

(i) The position of crack

(ii) The depth of crack

(iii)The orientation of crack

(iv)The number of cracks

3.2 CLASSIFICATION OF CRACKS

Based on their geometries, cracks can be broadly classified as follows:

• Cracks perpendicular to the beam axis are known as “transverse cracks”. These are the most common and most serious as they reduce the cross-section and thereby weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.

• Cracks parallel to the beam axis are known as “longitudinal cracks”. They are not that common but they pose danger when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.

• “Slant cracks” (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behavior of the beam. Their effect on lateral vibrations is less than that of transverse cracks of comparable severity.

• Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as “breathing cracks”. The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity’s in the vibration behavior of the beam. Cracks breathe when crack sizes are small, running speeds are low and radial forces are large .Most theoretical research efforts are concentrated on “transverse breathing” cracks due to their direct practical relevance.

• Cracks that always remain open are known as “gaping cracks”. They are more correctly called “notches”. Gaping cracks are easy to mimic in a laboratory environment and hence most experimental work is focused on this particular crack type.

• Cracks that open on the surface are called “surface cracks”. They can normally be detected by techniques such as dye-penetrates or visual inspection.

• Cracks that do not show on the surface are called “subsurface cracks”. Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than subsurface cracks on the vibration behavior of shafts.

CHAPTER-4

FREE VIBRATION OF CRACKED BEAM-COLUMNS

4. 1 DYNAMIC EQUATION FOR CRACKED BEAMS

The linear stability problem is accommodated in the FEM by the introduction of what is known as geometric stiffness which account for in plane loading on conventional bending stiffness, the effective stiffness vanishes at buckling load. This is an Eigen value problem with eigen values now being the critical values of loading magnitude at which buckling occurs usually lowest of these is of practical interest.

The second type of problem comes from the realm of structural dynamics. Alternative is restricted to the calculation of common structural components and forms .This requires the development of mass matrix which will represent the effect of dynamic loading (proportional to the square of frequency) which is set up during vibration. In common with Eigen values now represent the square of the natural frequency and Eigen vectors defining the deformed shape of the structure when vibrating at a particular natural frequency.

The equation of motion in matrix form for vibration of a beam under load is written as

$$[M]\{\ddot{q}\} + \lfloor [K] - P\lfloor K_g \rfloor \rfloor \{q\} = 0 \text{ --------------- (1)}$$

Where, $[M]$= Consistent mass matrix

$[K]$ = Bending stiffness matrix of the beam

$\lfloor K_g \rfloor$= Geometric stiffness matrix

$\{q\}$= Displacement vector

P = External force vector

For free vibration the equation (1) can be written as,

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \quad \text{------------------------- (2)}$$

Where, the forcing function, $P = 0$

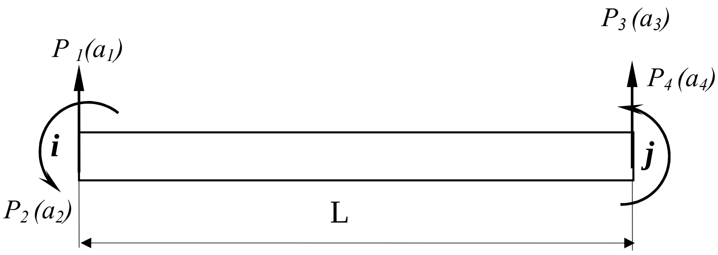
The equation (2) represents an Eigen value problem and the roots of the equation give rise to square of the natural frequency given by the equation,

The equation of motion in matrix form for buckling of a beam under load is written as

$$[K] - P_{cr}[K_G] = 0 \text{ -------------------- (3)}$$

4.2 FINITE ELEMENT ANALYSIS

In the present analysis two nodded beam element with two degrees of freedom (transverse deflection and slope) per node is considered and it is divided into 16 numbers of elements to get the result.



a_1 = deflection at node 1
a_2 = slope at node 1
a_3 = deflection at node 2
a_4 = slope at node 2
L = length of the element

Fig. 4.1 Intact beam element with 2 d.o.f per node

The displacement model taken as the polynomial as

$a(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3$ ---------------------- (4)

From this displacement model and putting the boundaries equation we arrive.

(1) At x=0; $a = \alpha_1$

$\therefore a_1 = \alpha_1$ ---------------------- (5)

At x=0; $a = \alpha_2$

$\therefore a_2 = \alpha_2$ ---------------------- (6)

(2) At $x = l; a = a_3$ and $\dot{a} = a_4$

$a_3 = a_1 + a_2 l + \alpha_3 l^2 + \alpha_4 l^3$ ---------------------- (7)

$a_4 = \alpha_2 + 2\alpha_3 l + 3\alpha_4 l^2$ ---------------------- (8)

From equations (4), (5), (6) and (7) we get

$\alpha_1 = a_1$;

$\alpha_2 = a_2$;

$$a_3 = -\frac{3}{l^2}\alpha_1 - \frac{2}{l}\alpha_2 + \frac{3}{l^2}\alpha_3 - \frac{\alpha_4}{l}$$

$$a_4 = \frac{2}{l^3}\alpha_1 + \frac{\alpha_2}{l^2} - \frac{2}{l^3}\alpha_3 + \frac{\alpha_4}{l^2}$$

Writing it in a matrix form

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

Then,

$$a = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3$$

$$a = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix}$$

$$a = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

So, $a = [N]\{a\}$ -- (9)

Where, $\{a\} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$

$[N] = [N_1 \quad N_2 \quad N_3 \quad N_4]$ ---------------------------------- (10)

Where

$$\left.\begin{aligned} N_1 &= 1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3} \\ N_2 &= x-\frac{2x^2}{l}+\frac{x^3}{l^2} \\ N_3 &= \frac{3x^2}{l^2}-\frac{2x^3}{l^3} \\ N_4 &= -\frac{x^2}{l}+\frac{x^3}{l^2} \end{aligned}\right]$$

The stress-strain relation of the beam-column element is nothing but the bending moment-curvature relationship.

$$M=-EI\frac{d^2w}{dx^2} \quad \text{---} \ (11)$$

Where, $w=[N]\{X_e\}$

Equ^n (11) written in matrix form for generality

$$\{\sigma\}=[D]\{\varepsilon\} \quad \text{--} \ (12)$$

Where,

$$\{\sigma\}=M,[D]=EI \text{ and } \{\varepsilon\}=-\frac{d^2w}{dx^2} \quad \text{---------} \ (13)$$

Differentiating both sides of equ^n (10) twice with respect to 'x' we get

$$\{\varepsilon\}=\left[\left(\frac{6}{l^2}-\frac{12x}{l^3}\right) \ \left(\frac{4}{l}-\frac{6x}{l^2}\right) \ \left(-\frac{6}{l^2}+\frac{12x}{l^3}\right) \ \left(\frac{2}{l}-\frac{6x}{l^2}\right)\right]\times\{X_e\} \quad \text{------------} \ (14)$$

$$(\varepsilon)=[B]\{X_e\} \quad \text{--} \ (15)$$

Combining equ^n (13) & (15) we get,

$$\{\sigma\}=[D]\{X_e\}[B] \quad \text{---------------------------------} \ (16)$$

Where, $[B]$ = strain displacement matrix

4.2.1 Elemental Stiffness matrix

The stiffness matrix for 2 degree of freedom *(v, θ)* for bending in the *xy*-plane for a two-noded Timoshenko beam finite element with shear deformation is line with Gounaris and Papazoglou [1992] as

$$[K] = \int_0^l [B]^T [D][B] \times dx \text{ --------------------------- (17)}$$

$$[K] = EI \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix} \text{ ------------------------ (18)}$$

Where, L = length of the element

E = young's modulus of elasticity

I = moment of inertia of the section with respect to *z*-axis

4.2.2 Elemental Mass matrix

$$[M] = \int_0^L [N]^T [\rho A][N] dx \text{ ---------------------------------- (19)}$$

$$[M] = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \text{ --------------- (20)}$$

Where, ρ = Mass density of the beam material

A = Cross-sectional area of the beam element

l = length of the element

4.2.3 Geometric Stiffness matrix

$$\left.\begin{aligned} N_1 &= 1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3} \\ N_2 &= x-\frac{2x^2}{l}+\frac{x^3}{l^2} \\ N_3 &= \frac{3x^2}{l^2}-\frac{2x^3}{l^3} \\ N_4 &= -\frac{x^2}{l}+\frac{x^3}{l^2} \end{aligned}\right]$$

$$G=\frac{d}{dx}[N]$$

$$[K_G]=\int_0^l [G]^T[G]P_{cr}\,dx \text{ --------------------------- (21)}$$

$$[K_G]=\frac{P_{cr}}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \text{ ------------------ (22)}$$

4.3 STIFFNESS MATRIX FOR A CRACKED BEAM ELEMENT

The key problem in using FEM is how to appropriately obtain the stiffness matrix for the cracked beam element. The most convenient method is to obtain the total flexibility matrix first and then take inverse of it. The total flexibility matrix of the cracked beam element includes two parts. The first part is original flexibility matrix of the intact beam. The second part is the additional flexibility matrix due to the existence of the crack, which leads to energy release and additional deformation of the structure.

4.3.1 Elements of the overall additional flexibility matrix C_{ovl}

The above figure shows a typical cracked beam element with a rectangular cross section. The left hand side end node ***i*** is assumed to be fixed, while the right hand side end node ***j*** is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2.

b = Breath of the beam

h = Depth of the beam

a = crack depth

L_c = Distance between the right hand side end node j and the crack location

Le = Length of the beam element

A = Cross-sectional area of the beam

I = Moment of inertia

According to Dimarogonas *et al.* (1983) and Tada *et al.* (2000) the additional strain energy due to existence of crack can be expressed as

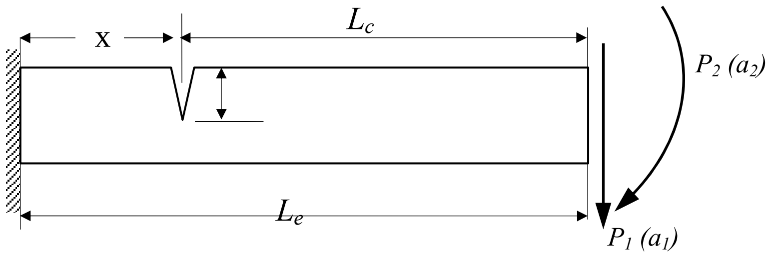


Fig. 4.2 Cracked beam element with 2 dof

$$\Pi_C = \int_{A_C} G dA_c \quad \text{-------------------------------} \ (23)$$

Where, G = the strain energy release rate and

A_C = the effective cracked area.

$$G = \frac{1}{E'}\left[\left(\sum_{n=1}^{2} K_{In}\right)^2 + \left(\sum_{n=1}^{2} K_{IIn}\right)^2 + k\left(\sum_{n=1}^{2} K_{IIIn}\right)^2\right] \quad \text{-----------------} \ (24)$$

Where, $E' = E$ for plane stress

$E' = E/1\text{-}\upsilon^2$ for plane strain

$k = 1 + \upsilon$

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks respectively

Neglecting effect of axial force and for open cracks, Eq.7 can be written as

$$G = \frac{I}{E'}\left[\left(K_{I1} + K_{I2}\right)^2 + K_{II1}^2\right] \quad \text{----------------- (25)}$$

The expressions for stress intensity factors from earlier studies are given by,

$$\left.\begin{aligned} K_{I1} &= \frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \\ K_{I2} &= \frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \\ K_{II1} &= \frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right) \end{aligned}\right] \quad \text{------------------------- (26)}$$

Where,

$$\left.\begin{aligned} F_I(s) &= \sqrt{\frac{\tan\left(\pi s/2\right)}{\left(\pi s/2\right)}}\left[\frac{0.923 + 0.199\left(1 - \sin\left(\pi s/2\right)\right)^4}{\cos\left(\pi s/2\right)}\right] \quad \left(s = \xi/h\right) \\ F_{II}(s) &= \frac{1.122 - 0.561s + 0.085s^2 + 0.180s^3}{\sqrt{1-s}} \quad \left(s = \xi/h\right) \end{aligned}\right] \quad \text{------------ (27)}$$

Here, $s = \frac{\xi}{h}$ (crack depth during the process of penetrating from zero to final depth), and $F_1(s)$ and $F_2(s)$ are the correction factors for stress intensity factors.

From definition, the elements of the overall additional flexibility matrix C_{ij} can be

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \prod_c}{\partial P_i \partial P_j} \quad (i,j = 1, 2) \quad \text{---------------------- (28)}$$

Substituting Eq. (26) into Eq. (25), then into Eq. (23) and Eq. (28) subsequently we get,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int\left[\left\{\frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)+\frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2+\left\{\frac{P_1}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right)\right\}^2\right]d\xi \text{ --- (29)}$$

Substituting i, j *(1, 2)* values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36L_c^2}{h^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx + \int_0^{\frac{a}{h}} xF_{II}^2(x)dx\right] \text{ --------------------- (30)}$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx = C_{21} \quad \text{-------------------- (31)}$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx \text{ ----------------------------- (32)}$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$C_{ovl} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \qquad \text{---------------------- (33)}$$

4.3.2 Flexibility matrix C_{intact} of the intact beam element

$$C_{\text{int}\,act} = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix} \text{ -------------------- (34)}$$

4.3.3 Total flexibility matrix C_{tot} of the cracked beam element

$$C_{total} = C_{intact} + C_{ovl}$$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI}+C_{11} & \frac{L_e^2}{2EI}+C_{12} \\ \frac{L_e^2}{2EI}+C_{21} & \frac{L_e}{EI}+C_{22} \end{bmatrix} \text{ ------------ (35)}$$

4.3.4 Stiffness matrix K_c of a cracked beam element:

From the equilibrium conditions, the stiffness matrix K_c of a cracked beam element can be obtained as

$$K_{crack} = LC_{tot}^{-1}L^{T} \text{ --------------------------- (36)}$$

Where L is the transformation matrix for equilibrium condition

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{--------------------- (37)}$$

The results are presented for vibration and buckling of beams with cracks using the present formulation. The boundary conditions are

- Fixed end: all degree of freedoms are constrained
- Free end: no restraint

4.3.5 Use of Gaussian Quadrature :

The numerical integration methods described so far are based on a rather simple choice of evaluation points for the function $f(x)$. They are particularly suited for regularly tabulated data, such as one might measure in a laboratory, or obtain from computer software designed to produce tables. If one has the freedom to choose the points at which to evaluate $f(x)$, a careful choice can lead to much more accuracy in evaluating the integral in question. We shall see that this method, called Gaussian or Gauss-Legendre integration, has one significant further advantage in many situations. This will prove valuable when evaluating various *improper* integrals, such as those with infinite limits.

The simplest form of Gaussian Integration is based on the use of an optimally chosen polynomial to approximate the integrand $f(t)$ over the interval [-1,+1]. The details of the determination of this polynomial, meaning determination of the coefficients of t in this polynomial, are beyond the scope of this presentation. The simplest form uses a uniform weighting over the interval, and the particular points at which to evaluate $f(t)$ are the roots of a particular class of polynomials, the Legendre polynomials, over the interval. It can be shown that the best estimate of the integral is then:

$$\int_{-1}^{1} f(t)dt = \sum_{i=1}^{n} w_i f(t_i)$$

Where,

t_i = a designated evaluation point

w_i = is the *weight* of that point in the sum

$f(t)$ = the function of the number of points at which is the value is evaluated is n.

Gaussian quadrature formulae are evaluating using abscissa and weights from a table (A) like that included here. The choice of value of n is not always clear, and experimentation is useful to see the influence of choosing a different number of points. When choosing to use n points, we call the method an ``n-point Gaussian" method. Here up to 6- Gaussian point is taken to get the accurate result.

4.3.6 Arguments and weighing factors for n-point Gauss Quadrature Rules:

In handbooks (see Table A), coefficients and arguments given for n-point Gauss Quadrature Rule are given for integrals of the form

$$\int_{-1}^{1} g(x)dx \cong \sum_{i=1}^{n} c_i g(x_i) \qquad \text{---------------------------- (38)}$$

TABLE-A

GAUSSIAN QUADRATURE FOR 6-POINT		
Points(n)	**Weight factor(C_i)**	**Function Arguments(x)**
6	+0.9324695142	0.1713244924
	-0.9324695142	0.1713244924
	+0.6612093865	0.3607615730
	-0.6612093865	0.3607615730
	+0.2386191861	0.4679139346
	-0.2386191861	0.4679139346

Put the 6-point Gaussian value from Table-(A) in Equ^n (46) and convert it into Gauss Quadrature equation as written in the above Equ^n (35),

$$F_I(s) = \sqrt{\frac{\tan\left(\frac{\pi s}{2}\right)}{\left(\frac{\pi s}{2}\right)}}\left[\frac{0.923 + 0.199\left(1-\sin\left(\frac{\pi s}{2}\right)\right)^4}{\cos\left(\frac{\pi s}{2}\right)}\right] \qquad \text{------------------ (39)}$$

CHAPTER-5

RESULTS AND DISCUSSION

5.1EXAMPLE PROBLEM

The method described has been applied to a cracked Bernoulli-Euler beam.

Properties:

Breath of the beam = 0.025 m

Depth of the beam = 0.0078 m

Length of the beam = 0.2 m

Elastic modulus of the beam = 216×10^9 N/m^2

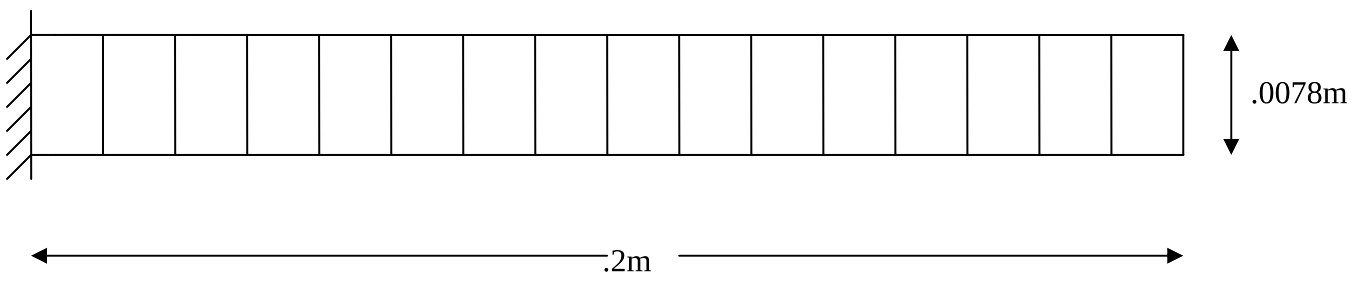
Poisson's Ratio = 0.28

Unit Weight = 7.85x103 kg/m^3

End condition of the beam = One end fixed and other end free (Cantilever beam).

The Finite element solutions are compared with previously established result in order to check the accuracy of the four lowest Eigen frequencies for various crack position and crack ratios. The beam was divided in to sixteen numbers of elements of equal size. It means the number of nodes is seventeen and considered two degrees of freedom of the beam.

Input of the program are taken as given in the above example .Output results for 10,12,14 and 16 elements are given in the table below which nearer to the M. Kisa , J Brandon and M. Topcu's paper.



(Fig-5.0 shows the element divided into 16 elements)

5.2 VALIDATION OF THE COMPUTED DATA WITH PREVIOUSLY ESTABLISHED RESULTS

Table: 5.1 Natural frequency of the cracked cantilever for Relative crack depth, at (x/L) = 0.2

Natural frequency	Present Analysis RCD 0.2 (rad/sec)	M.Kisa , et al. RCD 0.2 (rad/sec)	Present Analysis RCD 0.4 (rad/sec)	M.Kisa , et al. RCD 0.4 (rad/sec)	Present Analysis RCD 0.6 (rad/sec)	M.Kisa , et al. RCD 0.6 (rad/sec)	Present Analysis RCD 0.8 (rad/sec)	M.Kisa , et al. RCD 0.8 (rad/sec)	Intact beam Present Analysis (rad/sec)	Intact beam M.Kisa, et al. (rad/sec)
1st mode	1021.336	1020.137	967.3564	966.9525	842.4899	842.2205	551.3699	551.0463	1038.213	1037.0189
2nd mode	6500.996	6457.396	6496.141	6454.483	6487.744	6448.175	6472.6455	6436.008	6506.4079	6458.3438
3rd mode	18177.94	17872.91	17913.68	17596.57	17335.37	16944.56	16296.175	15512.550	18218.761	17960.564
4th mode	35137.88	34553.13	33955.41	33100.42	31878.46	29796.26	29376.157	25182.06	35705.479	34995.429

Table: 5.2 Natural frequency of the cracked cantilever for Relative crack depth, at (x/L) = 0.4

Natural frequency	Present Analysis RCD 0.2 (rad/sec)	M.Kisa , et al. RCD 0.2 (rad/sec)	Present Analysis RCD 0.4 (rad/sec)	M.Kisa , et al. RCD 0.4 (rad/sec)	Present Analysis RCD 0.6 (rad/sec)	M.Kisa , et al. RCD 0.6 (rad/sec)	Present Analysis RCD 0.8 (rad/sec)	M.Kisa , et al. RCD 0.8 (rad/sec)	Intact beam Present Analysis (rad/sec)	Intact beam M.Kisa, et al. (rad/sec)
1^{st} mode	1031.263	1030.0950	1007.917	1006.8560	943.529	942.7322	724.799	724.2739	1038.213	1037.0189
2^{nd} mode	6437.036	6389.3940	6220.939	6174.5390	5736.025	5689.8410	4784.72	4728.9780	6506.4079	6458.3438
3^{rd} mode	18096.734	17844.860	17735.868	17499.830	17033.485	16792.250	15975.641	15606.350	18218.761	17960.564
4^{th} mode	35594.008	34866.970	35243.092	34420.090	34519.796	32971.510	33234.558	29180.940	35705.479	34995.429

Table:5.3 Natural frequency of the cracked cantilever for Relative crack depth, at (x/L) = 0.6

Natural frequency	Present Analysis RCD 0.2 (rad/sec)	M.Kisa , et al. RCD 0.2 (rad/sec)	Present Analysis RCD 0.4 (rad/sec)	M.Kisa , et al. RCD 0.4 (rad/sec)	Present Analysis RCD 0.6 (rad/sec)	M.Kisa , et al. RCD 0.6 (rad/sec)	Present Analysis RCD 0.8 (rad/sec)	M.Kisa , et al. RCD 0.8 (rad/sec)	Intact beam Present Analysis (rad/sec)	Intact beam M.Kisa, et al. (rad/sec)
1st mode	1036.469	1035.2840	1030.405	1029.2620	1011.884	1010.8640	921.504	920.7848	1038.213	1037.0189
2nd mode	6412.481	6365.9140	6114.159	6071.6550	5409.167	5371.8030	3838.564	3798.2160	6506.4079	6458.3438
3rd mode	18067.616	17807.940	17625.788	17359.270	16782.861	16478.820	15580.145	15153.190	18218.761	17960.564
4th mode	35600.014	34895.500	35280.129	34572.370	34645.03	33710.43	33619.766	31412.070	35705.479	34995.429

Table: 5.4 Natural frequency of the cracked cantilever beam for Relative crack depth, at (x/L) = 0.8

Natural frequency	Present Analysis RCD 0.2 (rad/sec)	M.Kisa , et al. RCD 0.2 (rad/sec)	Present Analysis RCD 0.4 (rad/sec)	M.Kisa , et al. RCD 0.4 (rad/sec)	Present Analysis RCD 0.6 (rad/sec)	M.Kisa , et al. RCD 0.6 (rad/sec)	Present Analysis RCD 0.8 (rad/sec)	M.Kisa , et al. RCD 0.8 (rad/sec)	Intact beam Present Analysis (rad/sec)	Intact beam M.Kisa, et al. (rad/sec)
1st mode	1038.078	1036.8840	1037.605	1036.4140	1036.124	1034.9430	1027.922	1026.7690	1038.213	1037.0189
2nd mode	6487.608	6440.0570	6421.687	6375.9210	6215.806	6174.7100	5207.504	5169.2640	6506.4079	6458.3438
3rd mode	18008.390	17758.610	17304.268	17077.990	15492.768	15286.830	11627.445	11353.180	18218.761	17960.564
4th mode	35087.271	34393.870	33327.258	32639.520	30350.934	29529.790	27321.370	26230.830	35705.479	34995.429

5.3 VARIATION OF NATURAL FREQUENCY WITH RELATIVE CRACK DEPTH FOR(X/L = 0.2, 0.4, 0.6, 0.8)

Table: 5.5 (A) 1st Natural frequencies

RCD	X/L=0.2	X/L=0.4	X/L=0.6	X/L=0.8
0.2	1021.281	1031.263	1036.469	1038.078
0.4	967.906	1007.917	1030.405	1037.605
0.6	842.805	943.529	1011.884	1036.124
0.8	551.404	724.799	921.5042	1027.923

Table: 5.5(B) 2nd Natural frequencies

RCD	X/L=0.2	X/L=0.4	X/L=0.6	X/L=0.8
0.2	6505.679	6437.036	6412.481	6487.608
0.4	6500.685	6220.939	6114.159	6421.687
0.6	6485.634	5736.025	5409.167	6215.806
0.8	6430.403	4784.72	3838.564	5207.504

Table: 5.5(C) 3rd Natural frequencies

RCD	X/L=0.2	X/L=0.4	X/L=0.6	X/L=0.8
0.2	18131.39	18096.73	18067.62	18008.39
0.4	17854.31	17735.87	17625.79	17304.27
0.6	17239.32	17033.49	16782.86	15492.77
0.8	16076.3	15975.64	15580.15	11627.45

Table: 5.5(D) 4th Natural frequencies

RCD	X/L=0.2	X/L=0.4	X/L=0.6	X/L=0.8
0.2	35257.26	35594.01	35600.01	35087.27
0.4	33990.95	35243.09	35280.13	33327.26
0.6	31785.26	34519.8	34645.03	30350.93
0.8	29149.14	33234.56	33619.77	27321.37

5.4 COMPARISON OF MODESHAPES OF VIBRATION WITH INTACT BEAM FOR DIFFERENT RELATIVE CRACK DEPTH & CRACK LOCATION

Table: 5.6(A) Mode shape at x/L=0.2,RCD=0.2

	x/L=0.2, RCD=0.2					
	mode1	mode2	mode3	intact1	intact2	intact3
0	-3.44E-24	-7.73E-23	-3.58E-22	-3.56E-24	-7.75E-23	-3.56E-22
0.0125	-2.33E-02	-1.40E-01	-3.72E-01	-2.41E-02	-1.40E-01	-3.64E-01
0.025	-9.06E-02	-4.96E-01	-1.20E+00	-9.36E-02	-4.98E-01	-1.18E+00
0.0375	-1.98E-01	-9.79E-01	-2.10E+00	-2.04E-01	-9.84E-01	-2.04E+00
0.05	-3.47E-01	-1.51E+00	-2.65E+00	-3.52E-01	-1.51E+00	-2.62E+00
0.0625	-5.31E-01	-1.99E+00	-2.71E+00	-5.32E-01	-1.99E+00	-2.71E+00
0.075	-7.41E-01	-2.36E+00	-2.21E+00	-7.40E-01	-2.36E+00	-2.23E+00
0.0875	-9.76E-01	-2.57E+00	-1.26E+00	-9.74E-01	-2.57E+00	-1.28E+00
0.1	-1.23E+00	-2.58E+00	-5.22E-02	-1.23E+00	-2.58E+00	-7.12E-02
0.1125	-1.50E+00	-2.37E+00	1.13E+00	-1.50E+00	-2.37E+00	1.12E+00
0.125	-1.78E+00	-1.93E+00	2.01E+00	-1.78E+00	-1.93E+00	2.02E+00
0.1375	-2.08E+00	-1.30E+00	2.36E+00	-2.08E+00	-1.30E+00	2.38E+00
0.15	-2.38E+00	-4.87E-01	2.08E+00	-2.38E+00	-4.88E-01	2.10E+00
0.1625	-2.68E+00	4.50E-01	1.19E+00	-2.68E+00	4.49E-01	1.20E+00
0.175	-2.99E+00	1.47E+00	-1.88E-01	-2.99E+00	1.47E+00	-1.81E-01
0.1875	-3.30E+00	2.54E+00	-1.84E+00	-3.30E+00	2.54E+00	-1.85E+00
0.2	-3.61E+00	3.62E+00	-3.59E+00	-3.61E+00	3.61E+00	-3.61E+00

Table: 5.6(B) Mode shape at x/L=0.4,RCD=0.4

	x/L=0.4, RCD=0.4				(final)			
	mode1	mode2	mode3	mode4	intact1	intact2	intact3	intact4
0	-3.33E-24	-7.30E-23	-3.15E-22	-1.03E-21	-3.56E-24	-7.75E-23	-3.56E-22	-9.78E-22
0.0125	-2.27E-02	-1.39E-01	-3.31E-01	-6.98E-01	-2.41E-02	-1.40E-01	-3.64E-01	-6.59E-01
0.025	-8.81E-02	-4.99E-01	-1.08E+00	-1.99E+00	-9.36E-02	-4.98E-01	-1.18E+00	-1.88E+00
0.0375	-1.92E-01	-9.93E-01	-1.89E+00	-2.85E+00	-2.04E-01	-9.84E-01	-2.04E+00	-2.68E+00

0.05	-3.31E-01	-1.54E+00	-2.46E+00	-2.64E+00	-3.52E-01	-1.51E+00	-2.62E+00	-2.48E+00
0.0625	-5.01E-01	-2.06E+00	-2.62E+00	-1.33E+00	-5.32E-01	-1.99E+00	-2.71E+00	-1.26E+00
0.075	-6.97E-01	-2.49E+00	-2.29E+00	5.82E-01	-7.40E-01	-2.36E+00	-2.23E+00	4.71E-01
0.0875	-9.32E-01	-2.67E+00	-1.33E+00	2.05E+00	-9.74E-01	-2.57E+00	-1.28E+00	1.97E+00
0.1	-1.20E+00	-2.60E+00	-2.42E-02	2.48E+00	-1.23E+00	-2.58E+00	-7.12E-02	2.56E+00
0.1125	-1.48E+00	-2.34E+00	1.21E+00	1.86E+00	-1.50E+00	-2.37E+00	1.12E+00	1.99E+00
0.125	-1.77E+00	-1.88E+00	2.10E+00	4.50E-01	-1.78E+00	-1.93E+00	2.02E+00	5.26E-01
0.1375	-2.07E+00	-1.24E+00	2.44E+00	-1.12E+00	-2.08E+00	-1.30E+00	2.38E+00	-1.15E+00
0.15	-2.38E+00	-4.53E-01	2.13E+00	-2.13E+00	-2.38E+00	-4.88E-01	2.10E+00	-2.25E+00
0.1625	-2.69E+00	4.48E-01	1.19E+00	-2.08E+00	-2.68E+00	4.49E-01	1.20E+00	-2.22E+00
0.175	-3.01E+00	1.42E+00	-2.38E-01	-8.86E-01	-2.99E+00	1.47E+00	-1.81E-01	-9.63E-01
0.1875	-3.33E+00	2.44E+00	-1.94E+00	1.11E+00	-3.30E+00	2.54E+00	-1.85E+00	1.16E+00
0.2	-3.65E+00	3.47E+00	-3.74E+00	3.41E+00	-3.61E+00	3.61E+00	-3.61E+00	3.62E+00

Table: 5.6(C) Mode shape at x/L=0.6,RCD=0.6

	x/L=0.6, RCD=0.6					
	mode1	mode2	mode3	intact1	intact2	intact3
0	-3.34E-24	-5.26E-23	-3.49E-22	-3.56E-24	-7.75E-23	-3.56E-22
0.0125	-2.28E-02	-1.09E-01	-3.72E-01	-2.41E-02	-1.40E-01	-3.64E-01
0.025	-8.86E-02	-3.96E-01	-1.21E+00	-9.36E-02	-4.98E-01	-1.18E+00
0.0375	-1.93E-01	-7.96E-01	-2.13E+00	-2.04E-01	-9.84E-01	-2.04E+00
0.05	-3.33E-01	-1.25E+00	-2.79E+00	-3.52E-01	-1.51E+00	-2.62E+00
0.0625	-5.04E-01	-1.71E+00	-2.96E+00	-5.32E-01	-1.99E+00	-2.71E+00
0.075	-7.02E-01	-2.11E+00	-2.55E+00	-7.40E-01	-2.36E+00	-2.23E+00
0.0875	-9.23E-01	-2.42E+00	-1.59E+00	-9.74E-01	-2.57E+00	-1.28E+00
0.1	-1.16E+00	-2.60E+00	-2.67E-01	-1.23E+00	-2.58E+00	-7.12E-02
0.1125	-1.42E+00	-2.65E+00	1.19E+00	-1.50E+00	-2.37E+00	1.12E+00
0.125	-1.71E+00	-2.32E+00	2.15E+00	-1.78E+00	-1.93E+00	2.02E+00
0.1375	-2.03E+00	-1.52E+00	2.18E+00	-2.08E+00	-1.30E+00	2.38E+00
0.15	-2.36E+00	-6.09E-01	1.79E+00	-2.38E+00	-4.88E-01	2.10E+00
0.1625	-2.70E+00	3.97E-01	9.84E-01	-2.68E+00	4.49E-01	1.20E+00

0.175	-3.03E+00	1.46E+00	-1.45E-01	-2.99E+00	1.47E+00	-1.81E-01
0.1875	-3.37E+00	2.56E+00	-1.46E+00	-3.30E+00	2.54E+00	-1.85E+00
0.2	-3.71E+00	3.66E+00	-2.85E+00	-3.61E+00	3.61E+00	-3.61E+00

Table: 5.6(D) Mode shape at x/L=0.8,RCD=0.8

	x/L=0.8, RCD=0.8					
	mode1	mode2	mode3	intact1	intact2	intact3
0	-3.47E-24	-4.32E-23	-1.73E-22	-3.56E-24	-7.75E-23	-3.56E-22
0.0125	-2.36E-02	-9.00E-02	-2.28E-01	-2.41E-02	-1.40E-01	-3.64E-01
0.025	-9.16E-02	-3.26E-01	-0.7727	-9.36E-02	-4.98E-01	-1.18E+00
0.0375	-2.00E-01	-6.56E-01	-1.44E+00	-2.04E-01	-9.84E-01	-2.04E+00
0.05	-3.44E-01	-1.03E+00	-2.04E+00	-3.52E-01	-1.51E+00	-2.62E+00
0.0625	-5.21E-01	-1.40E+00	-2.42E+00	-5.32E-01	-1.99E+00	-2.71E+00
0.075	-7.25E-01	-1.73E+00	-2.50E+00	-7.40E-01	-2.36E+00	-2.23E+00
0.0875	-9.53E-01	-1.98E+00	-2.22E+00	-9.74E-01	-2.57E+00	-1.28E+00
0.1	-1.20E+00	-2.13E+00	-1.61E+00	-1.23E+00	-2.58E+00	-7.12E-02
0.1125	-1.47E+00	-2.16E+00	-7.09E-01	-1.50E+00	-2.37E+00	1.12E+00
0.125	-1.75E+00	-2.05E+00	3.61E-01	-1.78E+00	-1.93E+00	2.02E+00
0.1375	-2.03E+00	-1.82E+00	1.49E+00	-2.08E+00	-1.30E+00	2.38E+00
0.15	-2.33E+00	-1.47E+00	2.56E+00	-2.38E+00	-4.88E-01	2.10E+00
0.1625	-2.64E+00	-7.42E-01	2.93E+00	-2.68E+00	4.49E-01	1.20E+00
0.175	-3.01E+00	1.17E+00	8.25E-01	-2.99E+00	1.47E+00	-1.81E-01
0.1875	-3.37E+00	3.12E+00	-1.38E+00	-3.30E+00	2.54E+00	-1.85E+00
0.2	-3.74E+00	5.08E+00	-3.62E+00	-3.61E+00	3.61E+00	-3.61E+00

5.5 BUCKLING LOAD FOR DIFFERENT NO. OF ELEMENTS i.e.(10,12,14,16)

Table: 5.7(A) Buckling load for total no. of element 10

RCD	CBL (x/L=0.2)	CBL (x/L=0.4)	CBL (x/L=0.6)	CBL (x/L=0.8)	Intact
0.2	12802.18	12864.493	13086.6	13132.44	13172.741
0.4	11641.996	11881.071	12535.127	12705.05	13172.741
0.6	9015.115	9865.41	11847.31	11616.294	13172.741
0.8	3926.627	4466.227	7790.31	7317.142	13172.741

Table: 5.7(B) Buckling load for total no. of element 12

RCD	CBL (x/L=0.2)	CBL (x/L=0.4)	CBL (x/L=0.6)	CBL (x/L=0.8)	Intact
0.2	12808.28	12912.9	13045.418	13144.485	13172.741
0.4	11669.052	12068.45	12609.707	13044.3	13172.741
0.6	9091.29	9960.6	11352.388	12717.97	13172.741
0.8	4034.457	4916.298	6920.079	10661.219	13172.741

Table: 5.7(C) Buckling load for total no. of element 14

RCD	CBL (x/L=0.2)	CBL (x/L=0.4)	CBL (x/L=0.6)	CBL (x/L=0.8)	Intact
0.2	12805.223	12909.027	13037.03	13139.14	13172.741
0.4	11655.51	12055.54	12574.19	13020.12	13172.741
0.6	9053.049	9944.34	11251.109	12638	13172.741
0.8	3979.62	4937.66	6732.308	10420.39	13172.741

Table: 5.7(D) Buckling load for total no. of element 16

RCD	CBL (x/L=0.2)	CBL (x/L=0.4)	CBL (x/L=0.6)	CBL (x/L=0.8)	Intact
0.2	12803.857	12903.974	13029.149	13132.739	13172.741
0.4	11650.507	12035.990	12540.943	12991.180	13172.741
0.6	9043.417	9900.884	11157.913	12537.796	13172.741
0.8	3972.269	4892.735	6569.323	9969.254	13172.741

5.6 VARIATION OF BUCKLING LOAD WITH RELATIVE CRACK DEPTH AND SPAN LENGTH FOR (X/L = 0.2, 0.4, 0.6, 0.8)

Table: 5.8(A) Buckling load at X/L=0.2

			x/l=0.2				
RCD	l/d=20	l/d=25	l/d=30	l/d=35	l/d=40	l/d=45	l/d=50
0.2	20867.35	13458.63	9393.54	6925.976	5316.734	4209.459	3415.2
0.4	18487.274	12216.73	8665.08	6462.65	5316.733	3988.45	3988.45
0.6	13510.711	9429.95	6947.08	5328.762	4216.385	3419.245	3419.245
0.8	5312.268	4096.63	3263.11	2667.264	2225.651	1888.432	1888.432

Table: 5.8(B) Buckling load at X/L=0.4

			x/l=0.4				
RCD	l/d=20	l/d=25	l/d=30	l/d=35	l/d=40	l/d=45	l/d=50
0.2	21074	13566.45	9456.687	6966.077	5343.771	4228.52	3429.171
0.4	19255.36	12630.43	8913.316	6623.203	5113.757	4066.82	3311.2
0.6	15067.139	10341.96	7529.21	5723.48	4996.508	3625.23	2984.52
0.8	6683.085	5056.25	3969.91	3206.983	2649.12	2227.86	1901.368

Table: 5.8(C) Buckling load at X/L=0.6

			x/l=0.6				
RCD	l/d=20	l/d=25	l/d=30	l/d=35	l/d=40	l/d=45	l/d=50
0.2	21340.34	13701.6	9534.5	7014.901	5376.386	4251.398	3445.8
0.4	20293.913	13174.03	9232.52	6826.291	5250.839	4136.67	3382.11
0.6	17438.394	11684.37	8361.346	6274.186	4879.479	3902.161	3191.13
0.8	9226.334	6808.069	5237.11	4156.683	3380.548	2803.664	2362.941

Table: 5.8(D) Buckling load at X/L=0.8

			x/l=0.8				
RCD	l/d=20	l/d=25	l/d=30	l/d=35	l/d=40	l/d=45	l/d=50
0.2	21564.03	13813.54	9598.283	7054.613	5402.763	4269.77	3459.14
0.4	21255.03	13660.39	9511.594	7000.84	5367.206	4245.063	3441.248
0.6	20263.289	13169.51	9234.241	6829.315	5253.837	4166.308	3384.345
0.8	14906.589	10398.36	7636.556	5813.937	4592.406	3706.428	3052.262

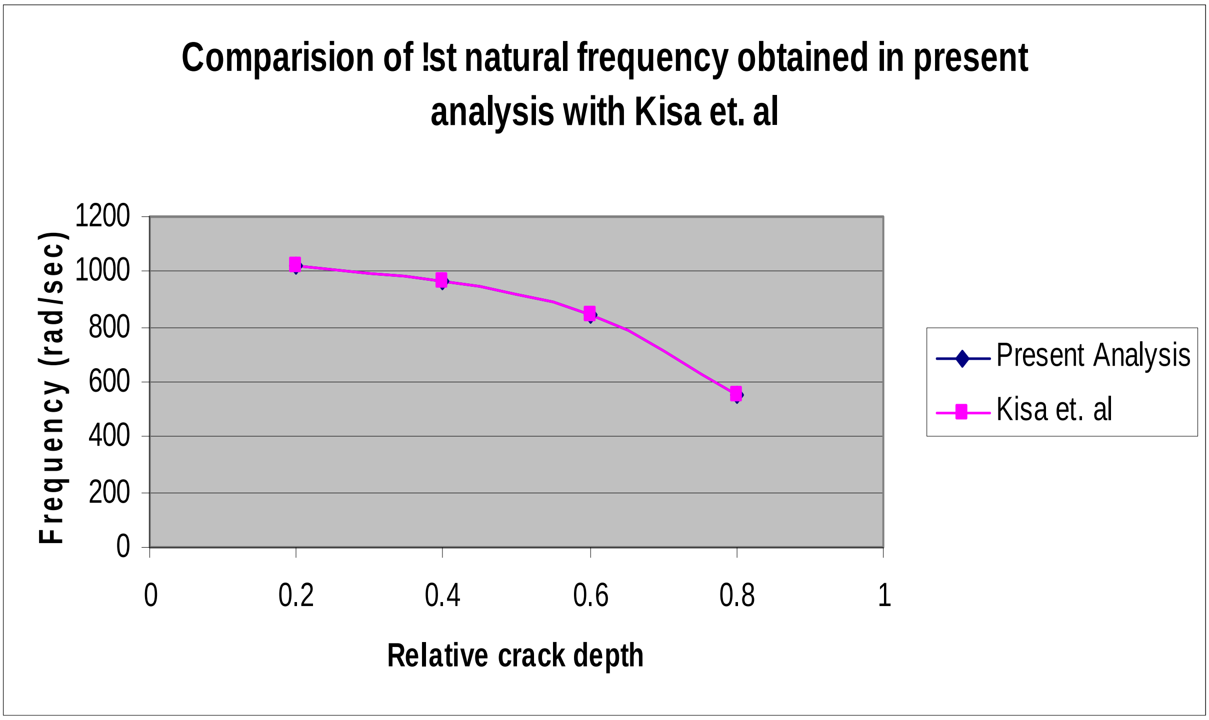


Fig: 5.1 1st Natural frequency vs Relative crack depth

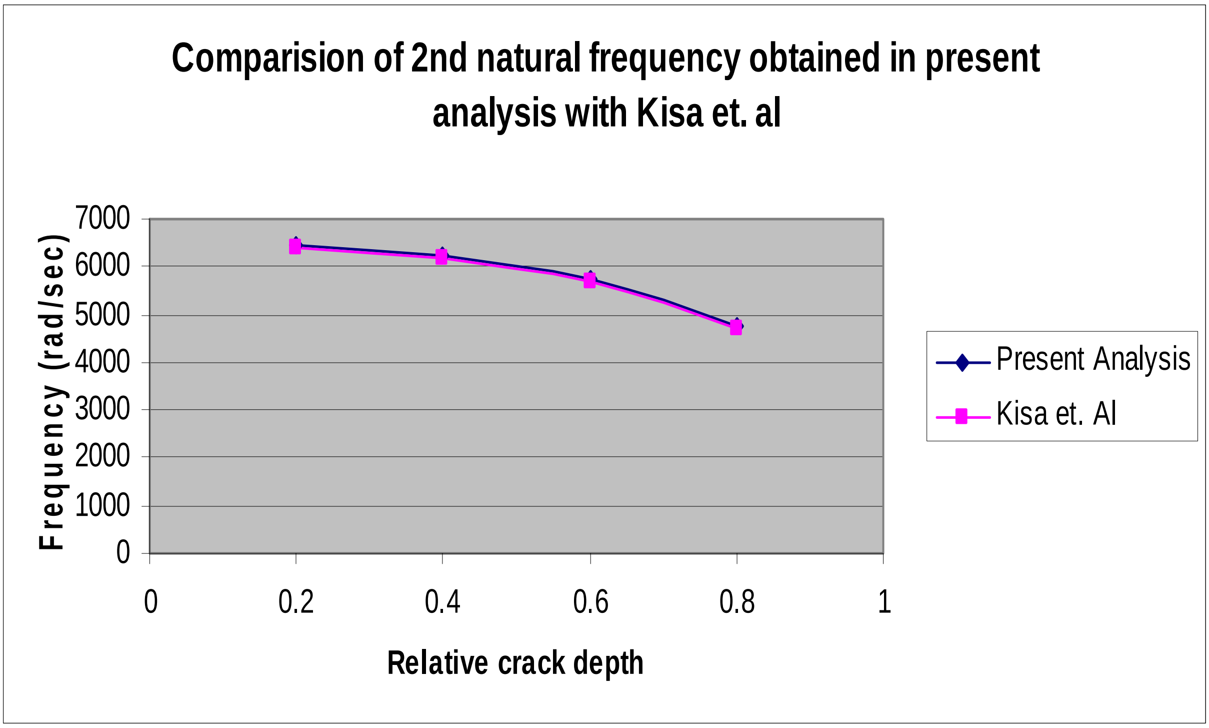


Fig: 5.2 2nd Natural frequency vs Relative crack depth

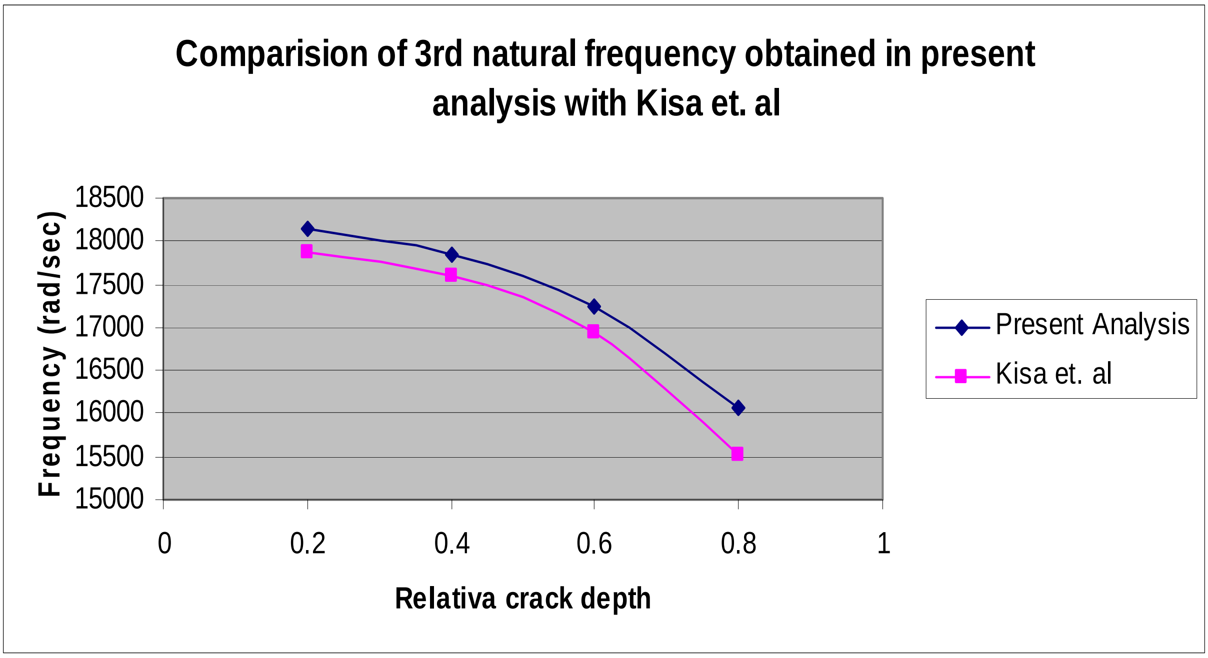


Fig: 5.3 3rd Natural frequency vs Relative crack depth

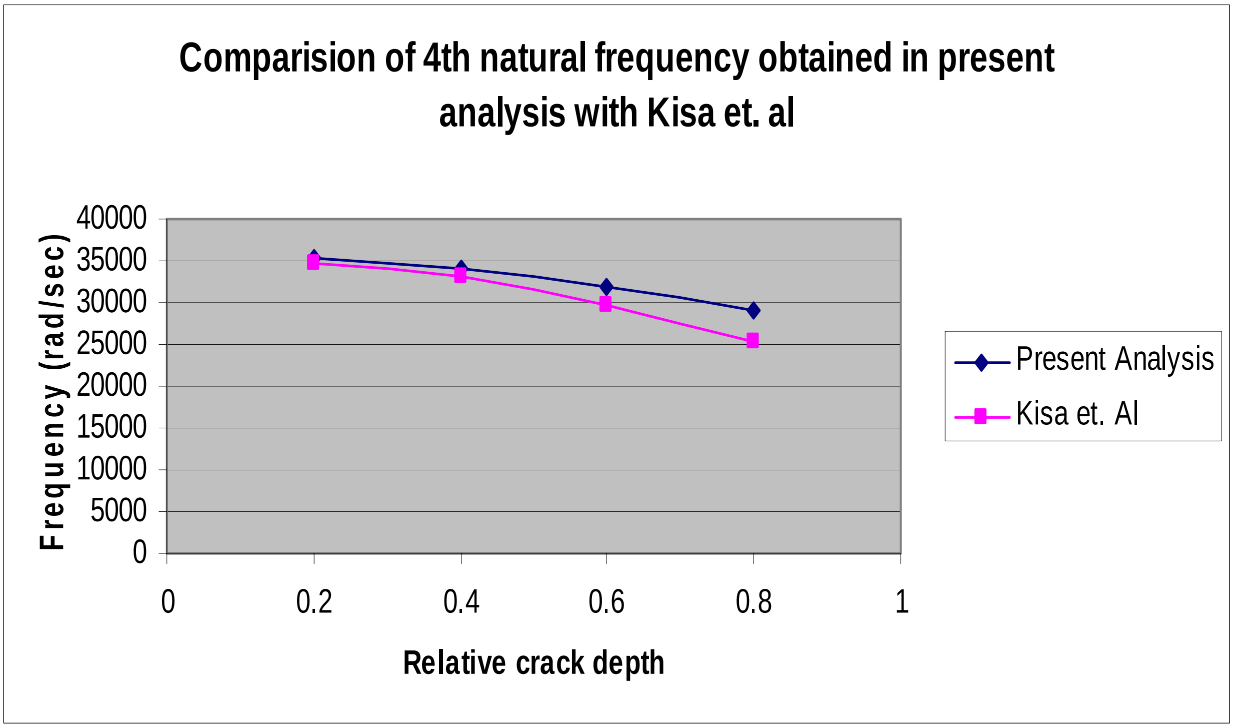


Fig: 5.4 4th Natural frequency vs Relative crack depth

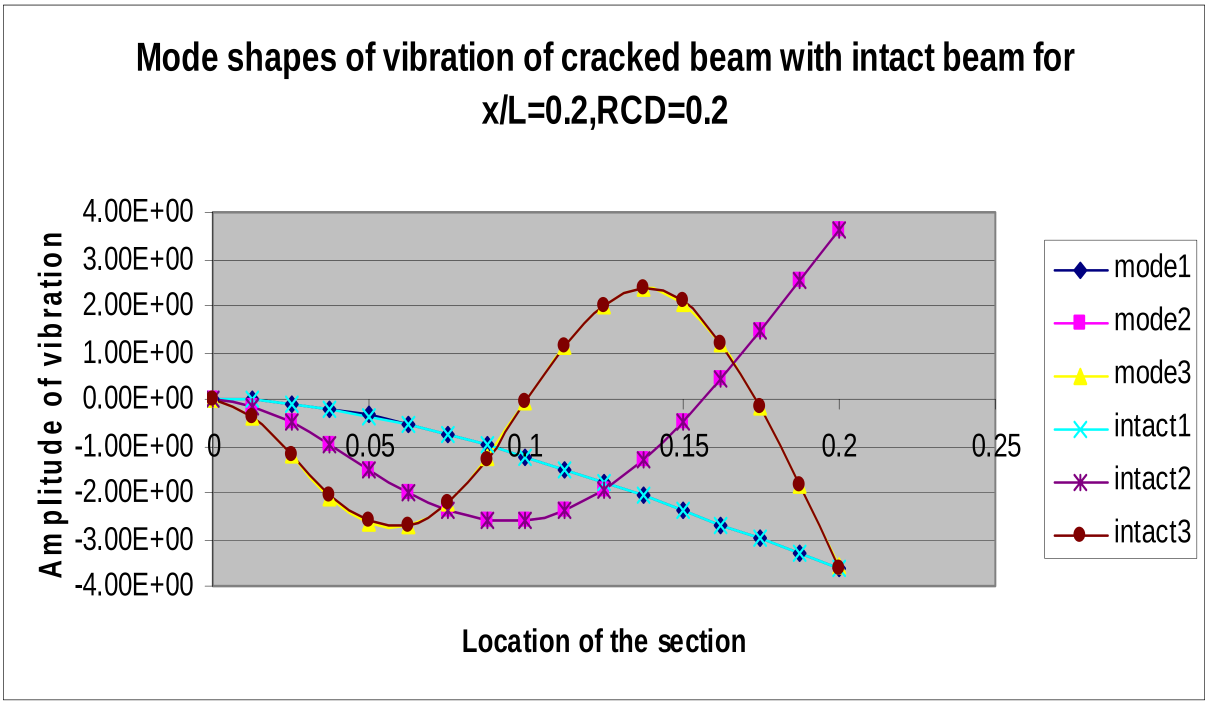


Fig: 5.5 Amplitude of vibration vs location of the section

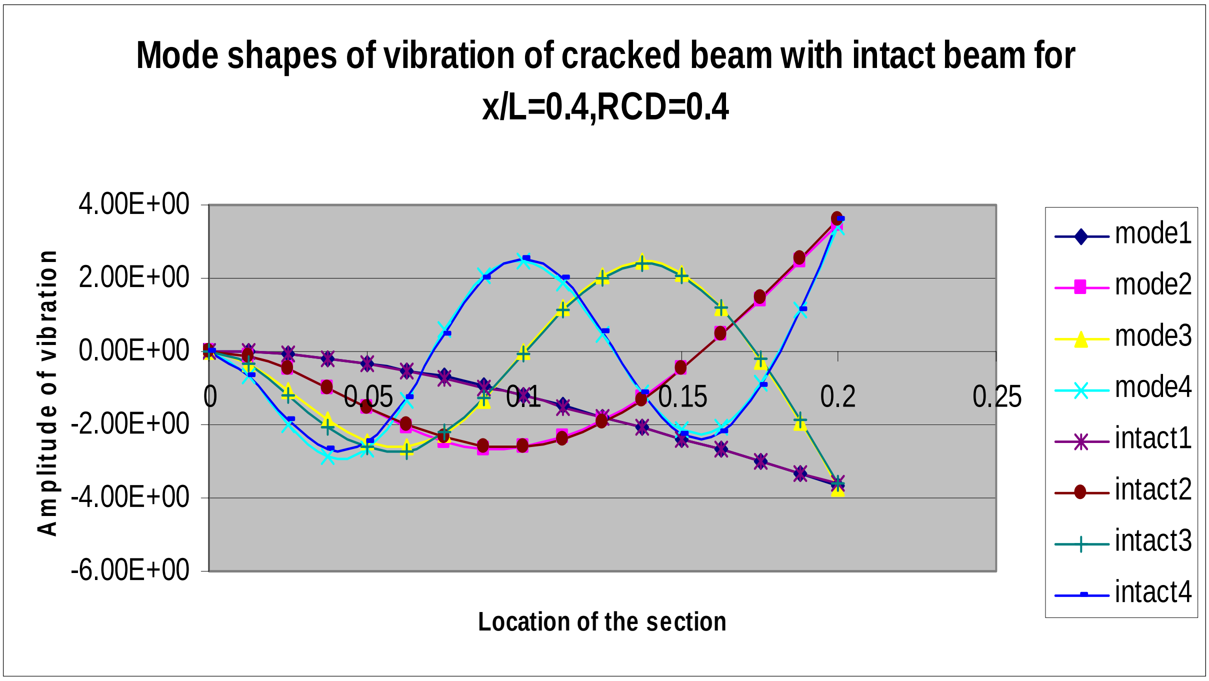


Fig: 5.6 Amplitude of vibration vs location of the section

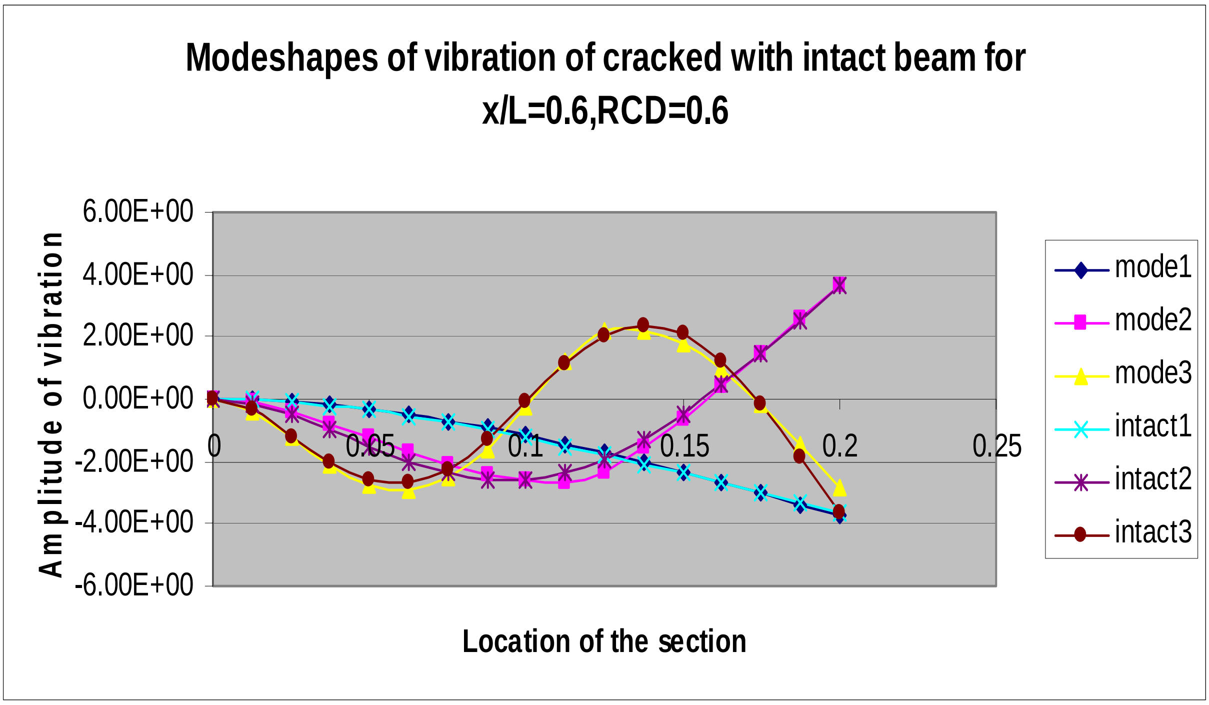


Fig: 5.7 Amplitude of vibration vs location of the section

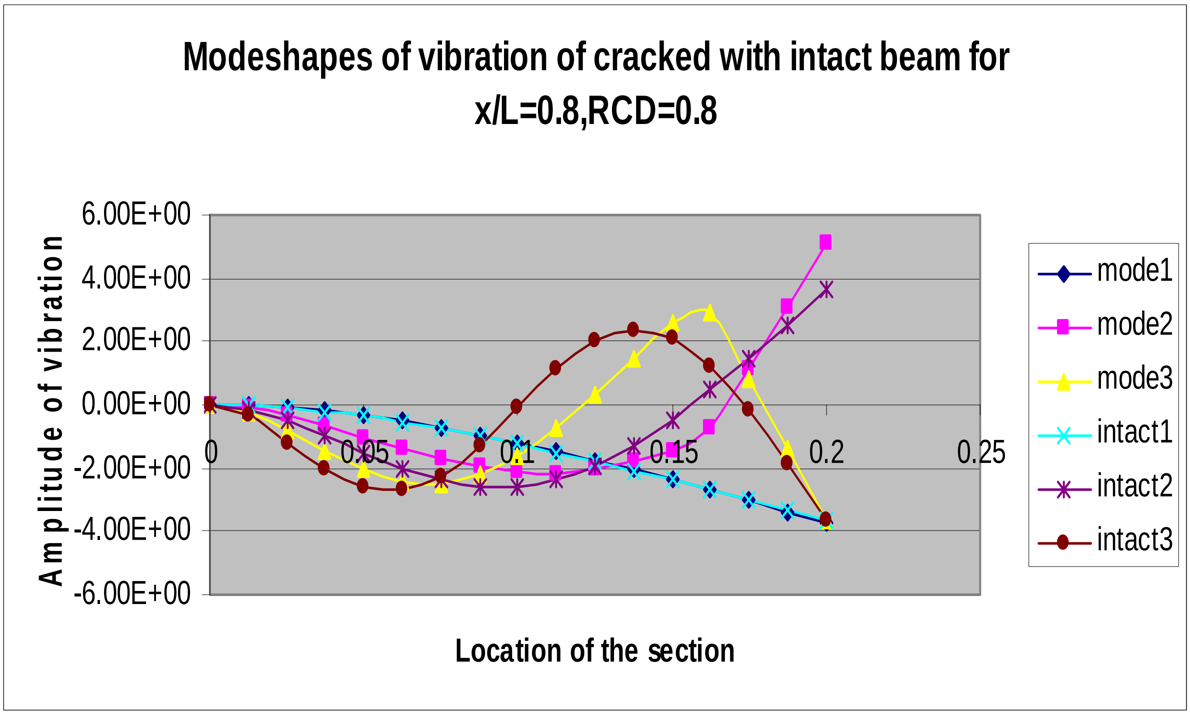


Fig: 5.8 Amplitude of vibration vs location of the section

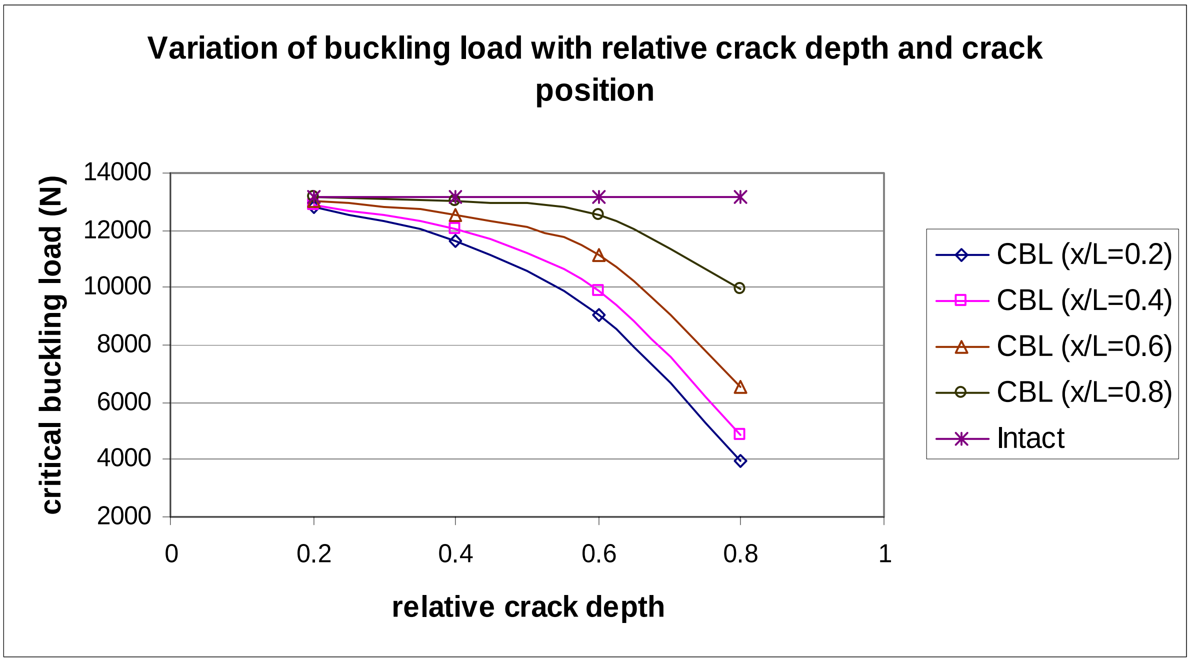


Fig: 5.9 Critical buckling load vs relative crack depth (for 16 no. of elements)

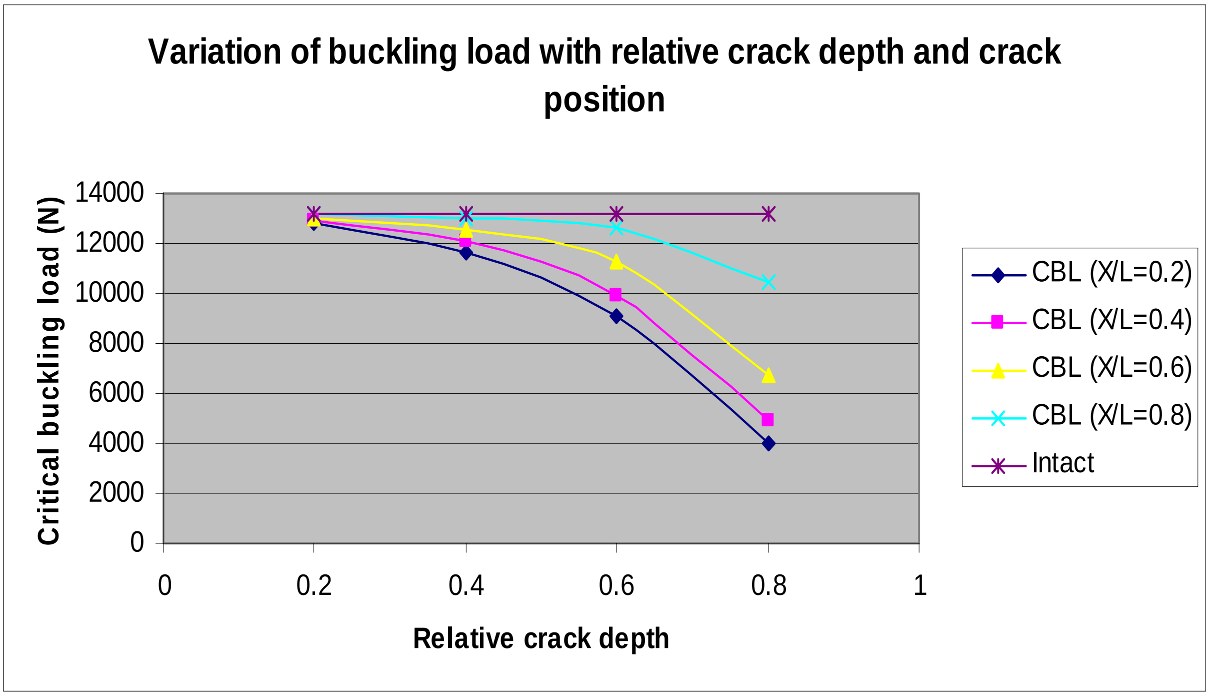


Fig: 5.10 Critical buckling load vs relative crack depth (for 14 no. of elements)

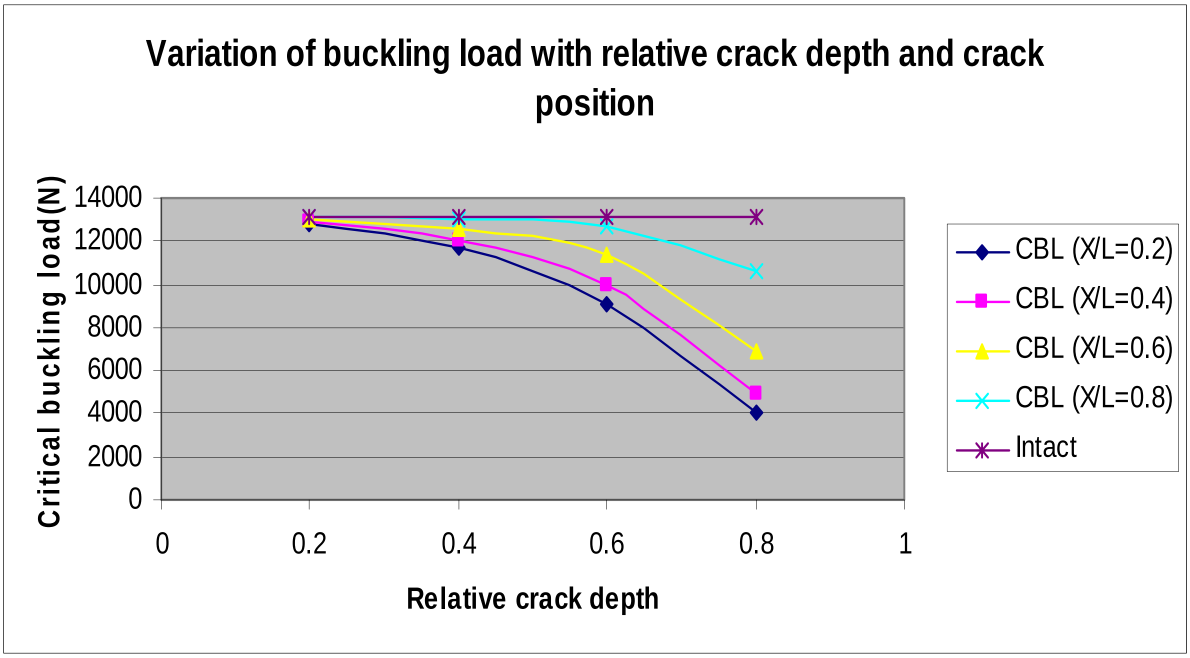


Fig: 5.11 Critical buckling load vs relative crack depth (for 12 no. of elements)

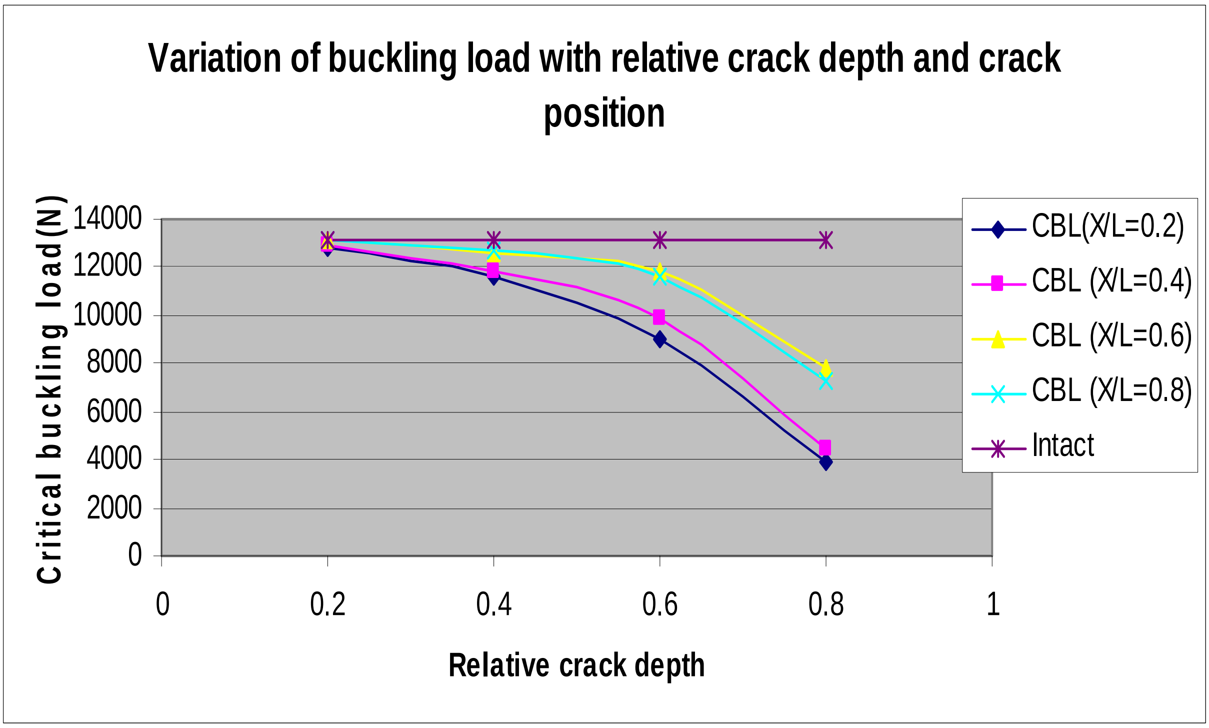


Fig: 5.12 Critical buckling load vs relative crack depth (for 10 no. of elements)

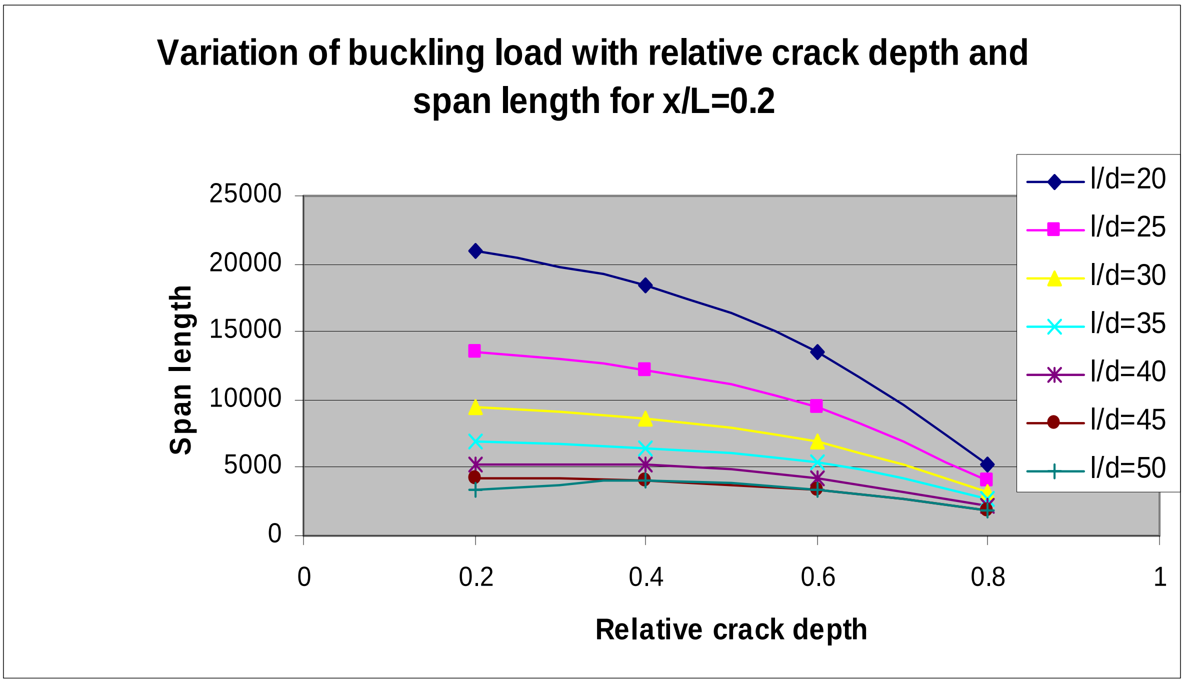


Fig: 5.13 Span length vs Relative crack depth

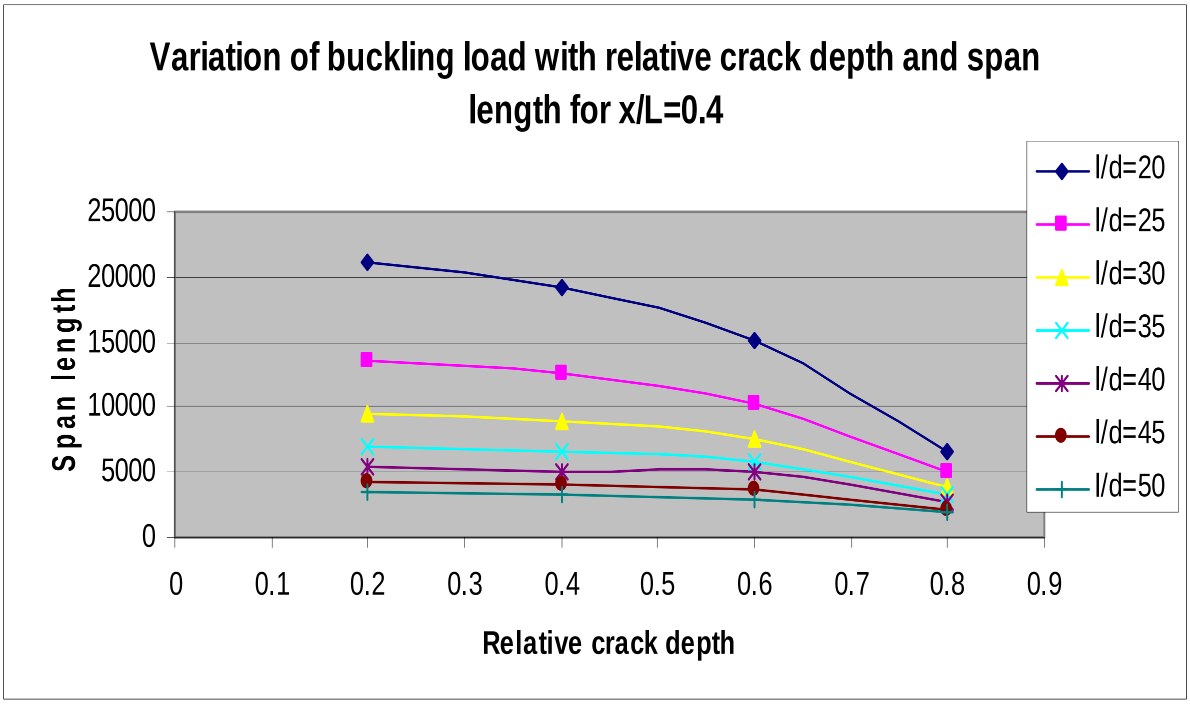


Fig: 5.14 Span length vs Relative crack depth

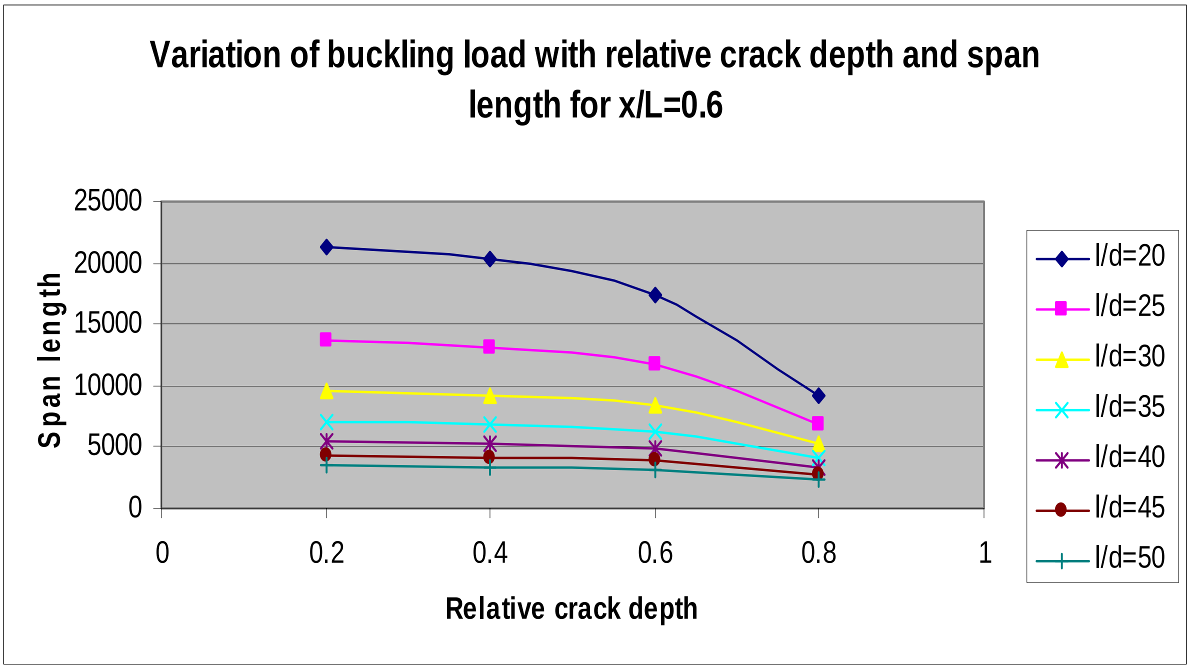


Fig: 5.15 Span length vs Relative crack depth

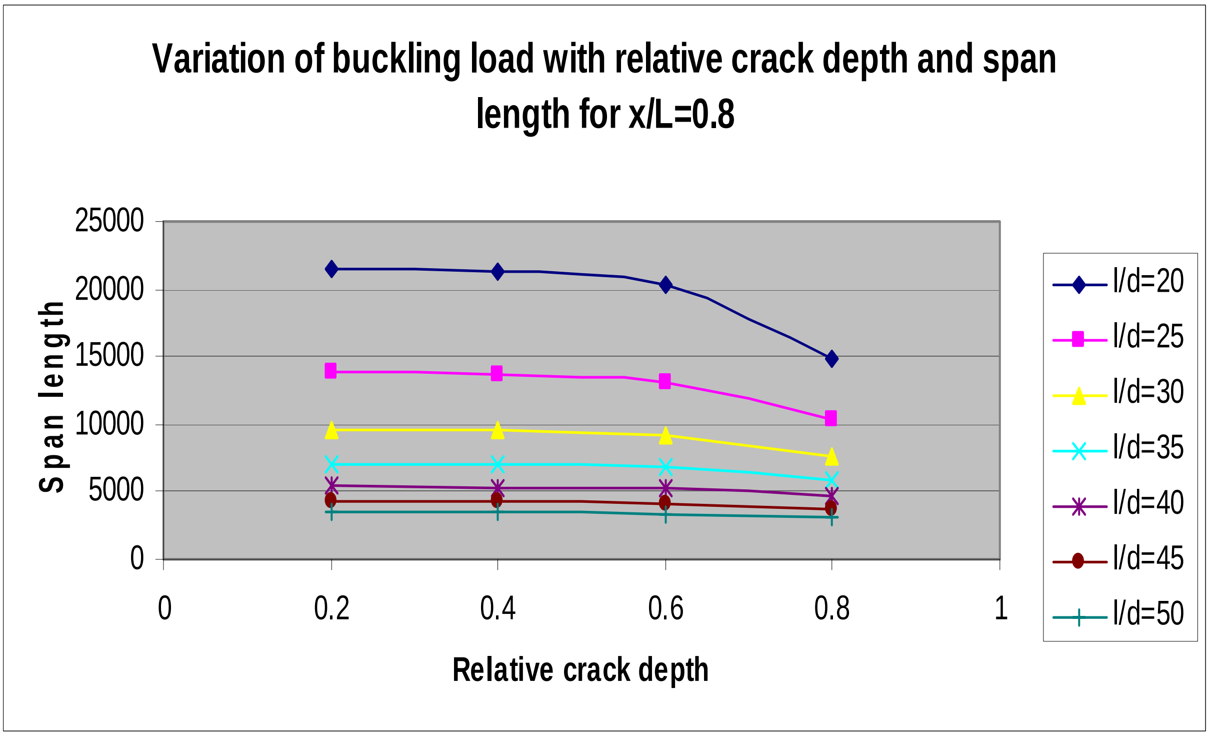


Fig: 5.16 Span length vs Relative crack depth

5.9 DISCUSSIONS

- The frequency of the cracked cantilever beam decreases with increase in the crack depth for the all modes of vibration.
- Natural frequencies of the cracked beam are lower than the natural frequencies of the corresponding intact beam as given in the TABLE (5.1) through TABLE (5.4). These differences increase with the depth of the crack.
- Due to bending moment along the beam, which is concentrated at the fixed end, a crack near the free end will have smaller effect on the fundamental frequency than a crack closer to the fixed end and it can be said that the frequencies are almost have a little difference at various relative crack depth when the crack is located away from the fixed end. As shown in TABLE-5.5 (A) through TABLE-5.5 (B).
- Amplitudes of Cracked beam-column compared with the Intact beam-column, when the crack is closer to the free end there is a little variation of amplitudes between them. As shown in TABLE-5.6 (A) through TABLE-5.6 (D)
- Critical buckling load of the cantilever beam decreases with increase in the relative crack depth (RCD) and Critical buckling load of a cracked beam-column is less than intact beam-column. As shown in TABLE-5.7 (A) through TABLE-5.7 (D).
- TABLE-5.8 (A) through TABLE-5.8 (D) shows variation of buckling load decreases when span length increases.
- (Fig: 5.1) to (Fig: 5.4) shows the comparison of natural frequencies with the existing paper and the result is almost equal; there is very little difference between the two.
- (Fig:5.5) to (Fig:5.8) shows the amplitude of cracked beam-column with different crack location and intact beam-column, the mode shapes of both are coincides with each other.
- Variation of buckling loads for different no. of elements are shown in (fig: 5.9) through (fig: 5.12). From the figure as the crack position decreases the buckling also decreases and as the crack position is nearer to the fixed end the bending is more compared to other crack locations.
- (Fig: 5.13) through (Fig: 5.16) shows the variation of buckling load with relative crack depth and span length for different crack position. Relative crack depth is 20% and for l/d = 20, then percentage of buckling is between 70%-60%. But when RCD is 80%, the buckling percentage is very less and it is considered to be very negligible.

CHAPTER-5

RESULTS AND DISCUSSION

6.1 CONCLUSIONS

- A method for identifying the crack location and depth of the uniform cantilever beam was developed by using the linear fracture mechanics theory .The finite element model of the cracked beam is constructed and used to determine its natural frequencies.
- From the FORTRAN program result it is observed that the natural frequency for a single cracked cantilever beam decreases as compared to the intact cantilever beam as expected.
- The frequency of the cracked cantilever beam decreases with increase in the crack depth for the all modes of vibration.
- Natural frequencies and mode shapes of the structure contain information about the location and dimensions of the damaged structure.
- Standard FEM procedure is followed which will lead to a generalized eigen value problem and thus natural frequencies, critical buckling loads are obtained.
- As the crack is away from the fixed end of the cantilever beam i.e. nearer to the free end, the Critical buckling load increases as shown from the result means reverse of the effect of frequency on the beam.
- It is shown that Gaussian Quadrature method can be used to get the value of $C_{11,}$ C12, C21 and C_{22}. Rather than using (128 x 128) point Gaussian Quadrature, we can use up to (6 x 6) Gaussian point which gives accurate results.
- In solving the buckling load of a cracked beam-column,only a standard linear eigen value equation needs to be solved. All the formulae are expressed in matrix form and therefore computer coding is straightforward.

6.2 SUGGESTED FUTURE WORK

- The cracked cantilever beam can be analyzed with multiple cracks
- The dynamic response of the cracked beams can be analyzed for different crack orientntation.
- Stability study of the cracked beams can be done with three degrees of freedom per node.
- Shear deformation can be taken into consideration for the analysis.

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APPENDIX: A

$$\text{F (1, 1)} = \int_0^{0.2} 2\times\pi\left[\frac{\tan\left(\pi x/2\right)}{\pi x/2}\times\frac{0.923+0.199\left(1-\sin\left(\pi x/2\right)\right)^4}{\cos\left(\pi x/2\right)}\right]dx \text{ --------------------- (a)}$$

$$\text{F (1, 1)} = \int_{-1}^{+1} 2\times\pi\times 0.1\left[2\frac{\tan\left(\frac{1}{2}\pi\left(0.1[t+1]\right)\right)}{\pi}\times\frac{0.923+0.199\left(1-\sin\left(\frac{1}{2}\pi\left(0.1[t+1]\right)\right)\right)^4}{\cos\left(\pi x/2\right)}\right]dx \text{ ---(b)}$$

Put t =-0.9324695142

$$I1 = 0.17132449242\left[2\pi\times 0.1\left[2\frac{\tan\left(\frac{1}{2}\pi[0.1(t+1)]\right)}{\pi}\right]\times\frac{\left[.752+2.02[(t+1)]+.37\times\left[1-\sin\left[\frac{1}{2}\pi[0.1(t+1)]\right]\right]^3\right]^2}{\cos\left[\frac{1}{2}\pi[0.1(t+1)]\right]^2}\right]$$

------(c)

Put all the 6-Gaussian points value in the above equation(c), find out upto I6 and add all these value from I1 to I6 that will be equal to the value of equation (b). It means that calculated values are correct and these values are matching with the out put value of FORTRAN code.

APPENDIX: B

$$F(1,1) = C_{11}E^{'}b = 2\pi\int_0^{a/h} xF_1^{\,2}(x)dx$$

$$F(1,2) = \frac{C_{12}E'bh}{L_c} = 12\pi\int_0^{a/h} xF_1(x)F_2(x)dx$$

$$F(1,3) = C_{13}E'bh = F(1,2)$$

$$F(2,2) = C_{22}E'b = 2\pi\left[\frac{36L_c^{\,2}}{h^2}\int_0^{a/h} xF_2^{\,2}(x)dx + \int_0^{a/h} xF_{II}^{\,2}(x)dx\right]$$

$$F(2,3) = \frac{C_{23}E'bh^2}{L_c} = 72\pi\int_0^{a/h} xF_2^{\,2}(x)dx$$

$$F(3,3) = C_{33}E'bh^2 = F(2,3)$$